Phys 410
Spring 2013
Lecture #22 Summary
13 March, 2013

We discussed several examples of constrained systems by the Lagrangian method. The key step is to identify the number of degrees of freedom in the problem and find the most efficient set of generalized coordinates. Examining the constraints in the system is often a good way to identify the appropriate generalized coordinates. Writing down the kinetic and potential energies in terms of these generalized coordinates is often facilitated by using Cartesian or cylindrical or spherical coordinates, and then converting completely to the generalized coordinates.

The first example was a particle of mass m trapped on a cylinder of radius R, but subjected to a Hooke's law restoring force ($\vec{F} = -k\vec{r}$) directed to a point on the axis of the cylinder. The generalized coordinates are the z-coordinate and the azimuthal angle φ. The two resulting Lagrange equations are uncoupled. The z equation gives simple harmonic oscillation, with solution $z(t) = A\cos(\omega_0 t - \delta)$, where $\omega_0 = \sqrt{k/m}$. The φ equation results in a statement of conservation of angular momentum $mR^2\dot{\varphi} = const$. Hence the particle rotates around the cylinder at a constant angular velocity while executing harmonic motion in the z-direction.

We then did the problem of a frictionless block sliding down the side of a wedge of angle α which is sliding horizontally over a frictionless surface. Because the block and wedge are constrained to remain in contact, and the wedge and horizontal surface are also constrained to remain in contact, there are really only two degrees of freedom in this problem: the displacement of the wedge in the horizontal direction (q_2), and the displacement of the block down the wedge (q_1). The kinetic and potential energies can be written in terms of these coordinates and their time derivatives. We found that the horizontal component of momentum is conserved, and that the block moves down the wedge with a constant acceleration that depends of the mass of the block and wedge, as well as the angle α. The time for the block to reach the bottom of the wedge is just that of a particle moving with constant acceleration.